

On a Traffic Control Problem

WEIZHEN GU

Texas State University, San Marcos, Texas
wg04@txstate.edu

XINGDE JIA

Texas State University, San Marcos, Texas
jia@txstate.edu

Abstract

A traffic network can be modeled by a digraph, while the traffic sensor location problem can be studied by using a minimum edge control set which is defined as follows. Let $G = (V, E)$ be a digraph. A subset F of the edge set E is called an edge control set of G if every flow of G is completely determined by F . An edge control set F is said to be minimum if $|F|$ is the least cardinality among all edge control sets of G . In this paper an algorithm for constructing a minimum edge control set of any digraph is established.

1. Introduction

Every year, many miles of new roads and highways are built and put in use in almost every urban area to accommodate the growing number of vehicles. However, the expansion of roads and highways still cannot keep up with the growth rate of the number of vehicles. More travel by more vehicles aggravates traffic congestion and causes multiple delays for drivers. According to a recent study by the Texas Transportation Institute [3], Americans spent 3.7 billion hours in travel delays in 2003, with a total cost of more than \$63 billion. On average, Americans spent 47 hours stuck in traffic in 2003, while ten years earlier they were spending 40 hours a year. Traffic congestion becomes one of the major obstacles for the further development of many urban areas, affecting millions of people's daily life. Constructing more roads and highways may improve the situation, but it is very costly, and in many cases it is impossible due to the existing structures.

The availability of the global positioning system (GPS) has a dramatic impact on the way people travel. Together with recent advances in computer and communication technologies it is now possible to build efficient traffic control systems to intelligently direct the traffic flow to its maximum capacity. Such systems will be the key toward solving the traffic congestion prob-

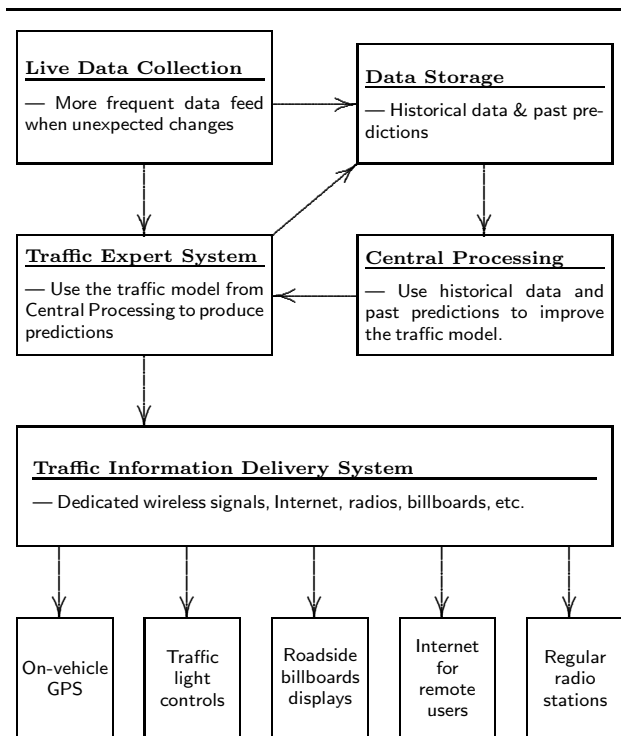


Figure 1. Intelligent Traffic Control System.

lem. As shown in Figure 1, an *intelligent traffic control system* (iTCS) consists of several key components. The *central processing* component uses the historical traffic information as well as the past predictions by the system to generate improved *traffic forecast models* which will be used by the *traffic expert system* to forecast traffic flow in the near future for the network. The traffic flow predictions will then be delivered to drivers via different channels such as roadside billboards, radio stations, the internet, and on-vehicle GPS systems. One key feature of the system is its ability to improve itself over time by learning from historical *real* traffic data and its past predictions.

One of the components of an iTCS is the live traffic data collection. In order to collect accurate traf-

fic data, sensors have to be placed on the streets and roads to measure the flow of traffic. One of the main questions in the collection of traffic data is where to place the traffic sensors. Of course one would like to use the least possible number of sensors to reduce cost and shorten the traffic data processing time.

A transportation network can be modeled by a digraph. In order to explain our results, we need the following definition.

Let G be a digraph. For each vertex v of G , let $E^+(v)$ (or $E^-(v)$) be the set of arcs with tail (or head) at v , i.e.,

$$\begin{aligned} E^+(v) &= \{uv \in E(G) : u \in V(G)\}, \\ E^-(v) &= \{uv \in E(G) : u \in V(G)\}. \end{aligned}$$

A function $f : E(G) \rightarrow \mathbb{R}^+ \cup \{0\}$ is called a *flow* of G if

$$\sum_{e \in E^+(v)} f(e) = \sum_{e \in E^-(v)} f(e) \quad \text{for all } v \in V(G) \quad (1)$$

A subset F of $E(G)$ is called an *edge control set* if, for any two flows f_1 and f_2 of G , it is always true that $f_1(e) = f_2(e)$ for all $e \in F$ implies that $f_1(e) = f_2(e)$ for all $e \in E(G)$.

If a traffic sensor is placed on each edge in an edge control set of the transportation network G , it follows from the definition of an edge control set that the sensors would provide complete traffic information for the network.

In this paper, we shall study the optimal locations for the traffic sensors. In Section 2, an algorithm for finding the optimal location for the traffic sensors is explained with an example, while its proof is given in Section 4. Section 3 discusses the minimum size of edge control sets for digraphs. We shall have some concluding remarks and open problems for further study at the end of this paper.

2. Minimal Edge Control Sets

An edge control set F is said to be *minimal* if any proper subset of F is not an edge control set of G . It is clear that minimal edge control sets are not unique.

In order to simplify the proofs, we will use a cut edge of a digraph for a cut edge of its underlying graph.

Definition 1 Let $G = (V, E)$ be a digraph, let H be a subgraph of G and $e \in E(H)$. Define

$$C_H(e) = \{e\} \cup \{d \in E(H) : d \text{ is a cut edge of } H - \{e\}\}.$$

We may call $C_H(e)$ the control of e in H .

Algorithm 2 Let G be a digraph such that each arc is on a directed cycle. A subset F is to be constructed by the following steps.

STEP 1. Let $F := \emptyset$ and $H := G$.

STEP 2. While $E(H) \neq \emptyset$, pick any edge $e \in E(H)$, let

$$F := F \cup \{e\}, \quad H := H - C_H(e).$$

Then F is a minimal edge control set of G .

We would like to point out that this algorithm can be applied to any digraph without the restriction that every arc is on a directed cycle, as discussed in the last section of this paper.

Before proving Algorithm 2, we use the digraph shown in Figure 2 to illustrate how an edge control set is constructed.

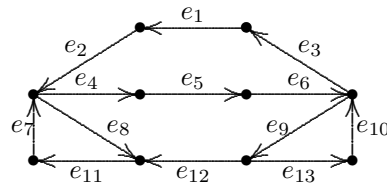


Figure 2. G is a digraph with each arc in a directed cycle.

We begin with $F = \emptyset$ and $H := G$ and then choose an edge for F , say e_1 . Then $F = \{e_1\}$, $C_H(e_1) = \{e_1, e_2, e_3\}$, and $H := H - C_H(e_1)$ shown in Figure 3.

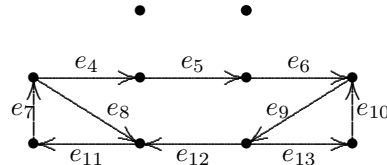


Figure 3. H is the first digraph obtained by applying Algorithm 2.

Since $E(H) \neq \emptyset$, we select e_4 and add it to F . So $F = \{e_1, e_4\}$, $C_H(e_4) = \{e_4, e_5, e_6, e_{12}\}$, and $H := H - C_H(e_4)$ displayed in Figure 4.

This time we add e_7 to F and then $C_H(e_7) = \{e_7, e_8, e_{11}\}$. So $H := H - C_H(e_7)$ consists of 7 isolated vertices and one cycle shown in Figure 5. Finally, we add e_9 into F . Therefore, $F = \{e_1, e_4, e_7, e_9\}$,

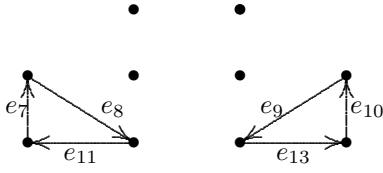


Figure 4. H is the second digraph obtained by applying Algorithm 2 .

$C_H(e_9) = \{e_9, e_{10}, e_{13}\}$, and H consists of all isolated vertices. In other words, $E(H) = \emptyset$. Therefore, $F = \{e_1, e_4, e_7, e_9\}$ is a minimal edge control set of G , according to Algorithm 2.

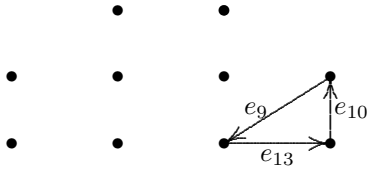


Figure 5. H is the third digraph obtained by applying Algorithm 2 .

3. Minimum Edge Control Sets

Let G be a digraph. Define by $\alpha(G)$ the minimum size of edge control sets of G , i.e.,

$$\alpha(G) = \min\{|F| : F \text{ is an edge control set of } G\}.$$

An edge control set F of G is called a *minimum edge control set* of G if $|F| = \alpha(G)$. A minimum edge control set of G must be a minimal edge control set of G .

Theorem 3 *Let G be a digraph with an edge control set F . Then F is a minimal edge control set of G if and only if F is a minimum edge control set of G .*

An immediate corollary of the theorem is as follows:

Corollary 4 *Let G be a digraph. If each arc of G lies on a directed cycle, then every edge control set produced by applying Algorithm 2 is a minimum edge control set of G .*

Theorem 5 (See [2]) *Let G be a digraph with $e \in E(G)$. Then e lies on a directed cycle of G if and only if there is a flow of G such that $f(e) \neq 0$.*

Lemma 6 *Let G be a digraph and let G^* be the digraph obtained from G by deleting all arcs that are not on any directed cycles of G . If F is a subset of $E(G)$, then F is a*

minimal edge control set of G if and only if F is a minimal edge control set of G^ .*

Proof. (\Leftarrow) Let F be a minimal edge control set of G^* . In order to show that F is also a minimal edge control set of G , we only need to show that F is an edge control set of G . Let f_1 and f_2 be two flows of G such that $f_1(e) = f_2(e)$ for all $e \in F$. Since $f_i(e) = 0$ for all $e \in E(G) - E(G^*)$, the restrictions of f_i on $E(G^*)$ are flows of G^* . Hence $f_1(e) = f_2(e)$ for all $e \in E(G^*)$. Therefore, $f_1(e) = f_2(e)$ for all $e \in E(G)$. That means that F is also an edge control set of G .

(\Rightarrow) Let F be a minimal edge control set of G . It follows from Theorem 5 that $F \subseteq E(G^*)$. It is easy to see that F is an edge control set of G^* . So we only need to prove the minimality of F . If not, there exists $F^* \subsetneq F$ such that F^* is an edge control set of G^* . As we proved earlier in this proof, F^* is an edge control set of G , which contradicts to the minimality of F as an edge control set of G . \square

Lemma 7 *Let G and G^* be defined as in Lemma 6. If F is a subset of $E(G)$, then F is a minimum edge control set of G if and only if F is a minimum edge control set of G^* .*

Proof of Theorem 3. (\Leftarrow) It follows from the definition of a minimum edge control set of G .

(\Rightarrow) Let G^* be the digraph obtained from G by deleting all arcs that do not lie on any directed cycles of G . Let F be a minimal edge control set of G . By Lemma 6 and Lemma 7, it is sufficient to show that F is a minimum edge control set of G^* .

Suppose that F is not a minimum edge control set of G^* . Let F^* be a minimum edge control set of G^* . Then

$$\alpha(G^*) = |F^*| < |F|.$$

By Lemma 7, F^* is a minimum edge control set of G . Thus $\alpha(G) = |F^*|$. Note that $|F| = \alpha(G)$. So

$$\alpha(G) = |F^*| < |F| = \alpha(G),$$

which is impossible. This completes the proof. \square

4. Proof of the Algorithm

Let X and Y be disjoint subsets of $V(G)$. We use $[X, Y]_G$ to denote the set of arcs with tails in X and heads in Y . In case of no confusion, we simply write $[X, Y]$ instead. $[X, Y]$ is called an *edge cut set* of G if X and Y form a partition of $V(G)$ such that $[Y, X] = \emptyset$. The following two useful lemmas are proved in [5, 2].

Lemma 8 *Let G be a digraph. If f is a flow of G , then for each partition (X, Y) of $V(G)$,*

$$\sum_{e \in [X, Y]} f(e) = \sum_{e \in [Y, X]} f(e).$$

Lemma 9 Let G be a digraph. If every arc of G lies on a directed cycle of G , then there exists a flow f of G such that $f(e) \neq 0$ for every $e \in E(G)$.

Lemma 10 Let G be a digraph such that each arc is on a directed cycle. Then the set F constructed by using Algorithm 2 is an edge control set of G .

Proof. Let $F = \{e_1, e_2, \dots, e_t\}$, where the edges being labeled in the same order as they are introduced into the set F by using Algorithm 2, and let

$$E(G) = E(H_0) \supset E(H_1) \supset \dots \supset E(H_t) = \emptyset$$

be the sequence of subgraphs generated by Algorithm 2. Then $H_i = H_{i-1} - C_{H_{i-1}}(e_i)$ for $i = 1, 2, \dots, t$. Let f_1 and f_2 be two flows of G with $f_1(e) = f_2(e)$ for all $e \in F$. We only need to show that $f_1(e) = f_2(e)$ for all the edges e of G . Suppose not, we choose the smallest integer τ so that $e_0 \notin H_\tau$ and $f_1(e_0) \neq f_2(e_0)$. Hence $f_1(e) = f_2(e)$ for all $e \in E(G - H_{\tau-1})$. Since $e_0 \notin F$, e_0 is a cut edge of $H_{\tau-1} - \{e_\tau\}$. Therefore, $\{e_0, e_\tau\}$ is the edge cut set of $H_{\tau-1}$. Assume that W' and W'' form a partition of $V(H_{\tau-1})$ with

$$\{e_0, e_\tau\} = [W', W'']_{H_{\tau-1}} \cup [W'', W']_{H_{\tau-1}}.$$

Define $V_1 = W'$ and $V_2 = V(G) - V_1$. Clearly V_1 and V_2 form a partition of $V(G)$ and $V_2 = W'' \cup V(G - H_{\tau-1})$. Let $E' = E_1 \cup E_2$, where

$$\begin{aligned} E_1 &= [V_1, V_2]_{G-H_{\tau-1}} \cup ([V_1, V_2]_G \cap \{e_0, e_\tau\}), \\ E_2 &= [V_2, V_1]_{G-H_{\tau-1}} \cup ([V_2, V_1]_G \cap \{e_0, e_\tau\}). \end{aligned}$$

Then E' is an edge cut set of G . It follows from Lemma 8 that

$$\sum_{e \in E_1} f_i(e) = \sum_{e \in E_2} f_i(e) \quad \text{for } i = 1, 2. \quad (2)$$

By the choice of e_0 and τ , we have $f_1(e) = f_2(e)$ for all $e \in E(G - H_{\tau-1}) \cup F$. Assume, without loss of generality, that $e_0 \in E_1$. It then follows from (2) that

$$\begin{aligned} & \sum_{e \in E_1} f_1(e) + \sum_{e \in E_2} f_2(e) \\ &= f_1(e_0) + \sum_{e \in E_1 - \{e_0\}} f_1(e) + \sum_{e \in E_2} f_2(e) \\ &= f_1(e_0) + \sum_{e \in E_1 - \{e_0\}} f_2(e) + \sum_{e \in E_2} f_1(e) \\ &= f_1(e_0) - f_2(e_0) + \sum_{e \in E_1} f_2(e) + \sum_{e \in E_2} f_1(e) \\ &= f_1(e_0) - f_2(e_0) + \sum_{e \in E_2} f_2(e) + \sum_{e \in E_1} f_1(e), \end{aligned}$$

which implies that $f_1(e_0) = f_2(e_0)$, contradicting the given assumption. Therefore, F is an edge control set of G . \square

Lemma 11 Let G be a digraph with subgraphs H_1 and H_2 such that $H_1 \subseteq H_2$. If H_1 and H_2 have no cut edges and $e \in E(H_1)$, then $C_{H_1}(e) \supseteq C_{H_2}(e)$.

Lemma 12 Let G be a digraph such that every arc lies on a directed cycle. Let $F = \{e_1, e_2, \dots, e_t\}$ be a minimal edge control set of G . Define

$$\begin{cases} H_0 &= G, \\ H_i &= H_{i-1} - C_{H_{i-1}}(e_i), \quad i = 1, 2, \dots, t. \end{cases} \quad (3)$$

then

- (i) For every $i : 1 \leq i \leq t$, H_i has no cut edges, i.e., every edge is on a cycle (not necessarily directed);
- (ii) $E(G) = E(H_0) \supseteq E(H_1) \supseteq \dots \supseteq E(H_t) = \emptyset$.

Proof. It follows directly from the definition of H_i that H_i has no cut edges. We now prove (ii).

First, we prove that the sequence in (ii) is indeed a strictly decreasing chain. We only need to show that

$$e_i \in E(H_{i-1}) \quad \text{for } i = 1, 2, \dots, t.$$

Suppose that $e_\mu \notin E(H_{\mu-1})$ for some $\mu : 1 \leq \mu \leq t$. Let ν be the smallest positive integer such that $e_\mu \in C_{H_{\nu-1}}(e_\nu)$. In other words, e_ν is the first arc in F such that $\{e_\mu, e_\nu\}$ is an edge cut set of $H_{\nu-1}$. We prove that $F - \{e_\mu\} = \bar{F}$ is an edge control set of G .

By a similar argument as in the proof of Lemma 10, we have that, for every flow f , $f(e_\mu)$ is completely determined by $f(e_i)$ ($i = 1, 2, \dots, \nu$). Therefore, $\bar{F} = F - \{e_\mu\}$ is an edge control set, contradicting the minimality of F .

We now prove $E(H_t) = \emptyset$. If $E(H_t) \neq \emptyset$, then we add more edges to F in the same way as in the algorithm. Assume that the following edges are added to F :

$$e_{t+1}, e_{t+2}, \dots, e_{t+s},$$

so that $E(H_{t+s+1}) = \emptyset$ and $E(H_{t+s}) \neq \emptyset$. It is clear that H_{t+s} consists of a cycle that is not necessarily directed, together with isolated vertices. Let the cycle in H_{t+s} be

$$C : v_1, v_2, \dots, v_k. \quad (4)$$

Let f be a flow of G such that $f(e) \neq 0$ for all $e \in E(G)$. The existence of such a flow follows from Lemma 9. Now we divide our proof according to whether or not C is a directed cycle.

CASE I: C is a directed cycle. Define $f_1 : E \rightarrow \mathbb{R}^+ \cup \{0\}$ by

$$f_1(e) = \begin{cases} f(e) + 1, & \text{if } e \in E(H_{t+s}), \\ f(e), & \text{otherwise.} \end{cases}$$

Since H_{t+s} does not contain any arcs in F , we have that $f(e) = f_1(e)$ for all $e \in F$. We now show that f_1 is also a flow of G . Let $v \in V(G)$. If $v \notin V(H_{t+s})$, then

$$\begin{aligned} \sum_{e \in E_G^+(v)} f_1(e) &= \sum_{e \in E_G^+(v)} f(e) \\ &= \sum_{e \in E_G^-(v)} f(e) = \sum_{e \in E_G^-(v)} f_1(e). \end{aligned}$$

If $v = v_i \in V(H_{t+s})$ for some $1 \leq i \leq k$, let $e' = v_{i-1}v_i$ and $e'' = v_iv_{i+1}$. Then

$$\begin{aligned} \sum_{e \in E_G^+(v)} f_1(e) &= \sum_{e' \neq e \in E_G^+(v)} f_1(e) + f_1(e') \\ &= \sum_{e' \neq e \in E_G^+(v)} f(e) + f(e') + 1 \\ &= \sum_{e \in E_G^+(v)} f(e) + 1 \\ &= \sum_{e \in E_G^-(v)} f(e) + 1 \\ &= \sum_{e'' \neq e \in E_G^-(v)} f(e) + f(e'') + 1 \\ &= \sum_{e \in E_G^-(v)} f_1(e). \end{aligned}$$

Therefore, f_1 is a flow of G . Recall that

$$f_1(e) = f(e) \quad \text{for } e \in F,$$

and

$$f_1(e) \neq f(e) \quad \text{for } e \in E(H_{t+s}).$$

This contradicts the fact that F is an edge control set of G .

CASE II: C is not a directed cycle. Recall that $C : v_1, v_2, \dots, v_k$. Assume that

$$v_{i_1}, v_{i_2}, \dots, v_{i_h}, \quad (5)$$

are the vertices of C that have degree in G larger than two, which are arranged in the same order as they are on the cycle (4). Since every arc is on a directed cycle, then the path of C between any two consecutive vertices in the above sequence (5) is directed. For the simplification of notation, we assume that the paths change directions at *every* vertex v_{i_j} , $j = 1, 2, \dots, h$. Note that h must be an even integer. Assume, without loss of generality, that the paths of C have the following indicated directions (see Figure 6).

$$\begin{aligned} P_1 &: v_{i_1} \rightarrow v_{i_2}, \\ P_2 &: v_{i_3} \rightarrow v_{i_2}, \\ &\vdots \\ P_{h-1} &: v_{i_{h-1}} \rightarrow v_{i_h}, \\ P_h &: v_{i_1} \rightarrow v_{i_h}. \end{aligned}$$

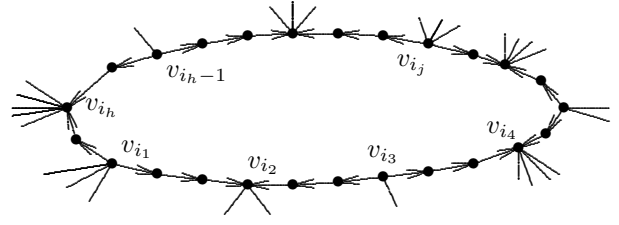


Figure 6. H_{t+s} is a cycle, together with isolated vertices (not showing).

Let $a = \min_{e \in E(G)} f(e)$, then $a > 0$. Define $f_1 : E \rightarrow \mathbb{R}^+ \cup \{0\}$ by

$$f_1(e) = \begin{cases} f(e) + (-1)^j a, & \text{if } e \in E(P_j) \forall j \\ f(e), & \text{otherwise} \end{cases}$$

Clearly, $f_1 \neq f$. We now prove that f_1 is also a flow of G . It follows from the definition of f_1 that we only need to prove that for every vertex v_i of H_{t+s} ,

$$\sum_{e \in E_G^+(v_i)} f_1(e) = \sum_{e \in E_G^-(v_i)} f_1(e).$$

Suppose that v_i is on the path P_j . If $\deg_G(v_i) = 2$, then v_i is an interior vertex of P_j . Let e' and e'' be the arcs incident with v_i . Since f is a flow of G , we have $f(e') = f(e'')$. Hence

$$f_1(e') = f(e') + (-1)^j a = f(e'') + (-1)^j a = f_1(e'').$$

If $\deg_G(v_i) > 2$, then we may assume that $v_i = v_{i_j}$ is the end vertices of directed paths P_{j-1} and P_j . Suppose that $e' \in E(P_{j-1})$ and $e'' \in E(P_j)$. We may assume, without loss of generality, that j is even. Then both e' and e'' are in $E_G^+(v_{i_j})$. Therefore,

$$\begin{aligned} \sum_{e \in E_G^+(v_{i_j})} f_1(e) &= \sum_{\substack{e \in E_G^+(v_{i_j}) \\ e \neq e', e''}} f_1(e) + f_1(e') + f_1(e'') \\ &= \sum_{\substack{e \in E_G^+(v_{i_j}) \\ e \neq e', e''}} f(e) + f(e') \\ &\quad + (-1)^{j-1} a + f(e'') + (-1)^j a \\ &= \sum_{e \in E_G^+(v_{i_j})} f(e) + (-1)^{j-1} a + (-1)^j a \\ &= \sum_{e \in E_G^-(v_{i_j})} f_1(e). \end{aligned}$$

Hence, f_1 is also a flow of G , which is a contradiction. Therefore, $E(H_t) = \emptyset$. The proof is complete. \square

Proof of Algorithm 2. It follows from Lemma 10 that the set F constructed in algorithm is an edge control set of G . We now only need to prove that F is minimal.

Assume that $F = \{e_1, e_2, \dots, e_t\}$, where the arc are labeled in the same order as they are introduced into the set F . If the edge control set F is not minimal, then let s be the smallest positive integer such that the proper subset $F - \{e_s\}$ is also an edge control set of G . It follows from the construction in Algorithm 2 that $e_s \in E(H_{s-1})$ and

$$e_s \notin E(H_s) = E(H_{s-1} - C_{H_{s-1}}(e_s)).$$

Let $F_1 = \{e_1, e_2, \dots, e_{s-1}, e_{i_s}, e_{i_{s+1}}, \dots, e_{i_k}\}$ be a minimal edge control set of G , where

$$s + 1 \leq i_s < i_{s+1} < \dots < i_k \leq t.$$

For convenience, let $i_j = j$ for $j = 1, 2, \dots, s-1$. Define

$$\begin{aligned} \tilde{H}_0 &= G, \\ \tilde{H}_j &= \tilde{H}_{j-1} - C_{\tilde{H}_{j-1}}(e_{i_j}), \quad j = 1, 2, \dots, k. \end{aligned}$$

Obviously, we have

$$\begin{aligned} \tilde{H}_j &= H_j \quad \text{for } j = 1, 2, \dots, s-1, \\ \tilde{H}_j &\supseteq H_{i_j} \quad \text{for } j = s, s+1, \dots, k. \end{aligned}$$

Since F_1 is a minimal edge control set of G , it follows from Lemma 12 that $E(\tilde{H}_k) = \emptyset$. Let λ be the smallest integer such that $e_s \notin E(\tilde{H}_\lambda)$. Then $e_s \in E(\tilde{H}_{\lambda-1})$, which implies $e_s \in C_{\tilde{H}_{\lambda-1}}(e_{i_\lambda})$. Therefore, $\{e_s, e_{i_\lambda}\}$ is an edge cut set of $\tilde{H}_{\lambda-1}$. Since $e_{i_\lambda} \in H_{i_\lambda-1}$ and $\tilde{H}_{\lambda-1} \supseteq H_{i_\lambda-1} \supseteq H_{i_\lambda-1}$, a subset of $\{e_s, e_{i_\lambda}\}$ is also an edge cut set for $H_{i_\lambda-1}$. Since $e_s \in E(H_{s-1}) = E(\tilde{H}_{s-1})$, it follows from the definition of λ that $\lambda \geq s$. Hence $i_\lambda - 1 \geq s$. Noting that $e_s \notin E(H_s)$ and $E(H_s) \supseteq E(H_{i_\lambda-1})$, we have $e_s \notin E(H_{i_\lambda-1})$. It follows that e_{i_λ} is a cut edge of $H_{i_\lambda-1}$, a contradiction to the construction in the algorithm. Therefore, F is a minimal edge control set of G . The proof is complete. \square

5. Remarks and Open Problems

The following are some remarks and open problems for further study.

1. Let G be a digraph and f a flow on G . It is proved in Theorem 5 that $f(e) = 0$ if e is not on any directed cycle. Therefore, Algorithm 2 can be used to find a minimal edge control set for G by applying it to the digraph obtained by deleting all edges of G that are not on any directed cycle.
2. Let G be a planar digraph with m edges and n vertices. It is proved [2] that a minimum edge control set of G contains $m - n + 1$ edges. Is this true for any digraph?

3. We have assumed in this paper that the traffic flow within the network contains neither *sinks* nor *sources*, namely,

$$\sum_{e \in E^+(v)} f(e) = \sum_{e \in E^-(v)} f(e) \quad \text{for all } v \in V(G).$$

For flows of digraphs without the above condition, modify the concept of the minimum edge control set, and establish an algorithm for finding such modified minimum edge control sets.

4. The traffic forecast model used by the central processing component of the iTCS will be studied in a separate research.
5. The results in this paper is more suitable for monitoring traffic flow instead of the speed of the traffic.

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References

- [1] G. Chartrand and P. Zhang, *Introduction to Graph Theory*, The Walter Rudin Student Series in Advanced Mathematics, McGraw Hill, Inc., 2005.
- [2] W. Gu and X. Jia, On Traffic Sensing Problem, Preprint, 2004.
- [3] Texas Transportation Institute, *The 2005 Urban Mobility Report*, May 2005.
- [4] D. B. West, *Introduction to Graph Theory*, 2nd Ed., Prentice Hall, Inc., New Jersey, 2001.
- [5] C.-Q. Zhang, *Integer Flows and Cycle Covers of Graphs*, Monographs and Textbooks in Pure and Applied Mathematics **205**, Marcel Dekker, Inc., 1997.